Limits of applicability of Riemannian geometry

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The current theories of physics, namely, the special and general theory of relativity, quantum mechanics and the quantum theory of fields have been formulated on the fabric of a continuous spacetime. By continuous spacetime, we mean a point in spacetime described by four coordinates (ct, x, y, z) that can take values varying continuously. The question that may arise immediately to the inquisitive mind is whether the concept of a continuous spacetime is always valid. It is important to realize that asking this question is in itself a giant leap from our common sense. Common sense would tell us that spacetime around us must be continuous because a discrete structure of spacetime would mean that there are regions (if at all they can be called as regions) where there is no spacetime at all.

But our common sense or intuition is well known to work well at length scales larger than some standard of length. A natural standard of length (spatial distance) would be the depth of a swimming pool, for instance, a metre. What about moving to smaller and smaller length scales and entering the quantum world. Our current understanding of the quantum domain is counter intuitive. Can we be really sure that on a "small scale", time and space do not become quite different from our present notion ? Spacetime may well become fragmentized at very small length scales. Such a remarkable idea or question about the structure of spacetime is not novel at all. In fact, it was raised by Riemann himself in 1854, and thereafter, it has been discussed repeatedly. However, this extremely important question about the limits of applicability of Riemannian geometry is still far from being answered. To understand the problem in a simple way, let us imagine looking at a piece of cloth from some distance apart. What we shall observe is a continuous fabric of cloth from a distance of say two feet from the cloth. However, as we move nearer and nearer to the cloth, we shall start observing the threads with which the cloth has been woven. In fact, we shall observe regions where there are no threads of the cloth at all, and the very notion of a continuos fabric of cloth breaks down. A similar situation is expected to happen also in the case of spacetime at energies close to the Planck energy, which in turn implies length scales close to the Planck length.

In the recent years, this problem about the status of Riemannian geometry at very small length scales has been one of the major concerns of theoretical high energy physicists. The reason for this concern lies in the search of a unified physical theory which would incorporate all the fundamental interactions. It turns out that as we move to the regime of increasingly high energies, and therefore, to closer collisions between various particles, the scale of unexplored space regions becomes smaller. Our present understanding indicates that the usual space relations down to the distance of the order of 10^{-15} cm goes through in the usual Riemannian way and their applications do not lead to inconsistencies. As we keep on increasing the energy of collision of particles further, the distance probed would be smaller and smaller, and it is highly plausible that there exists a fundamental length scale $l_p = \sqrt{\frac{G\hbar}{c^3}} = 10^{-33}$ cm. This fundamental length is known as the Planck length. If a fundamental length scale does exist, it would definitely tell theoretical physicists and mathematicians to reformulate our standard conception of a continuous spacetime fabric. It would also mean that Einstein's theory of general relativity, which is essentially a dynamical theory of spacetime formulated on the foundations of Riemannian geometry would possibly break down at distances close to the Planck length. In technical terms one would say that Einstein's theory of gravity has to be quantized. In reality, it would be a daunting task to write down a theory of gravity at length scales close to the Planck length, if the very notion of a continuous fabric of spacetime breaks down at such length scales. Such a theory, if at all can be formulated, would need to pass certain tests, for example, it would need to predict something new which has not been observed earlier, and also need to yield some signatures of a discrete spacetime in the low energy limit which can be tested experimentally. Efforts to formulate such a quantum Einstein gravity theory have been going on for the last six decades and the truth is still far from reality. One of the major hindrances in developing a quantized version of gravity is that Einstein's theory of gravitation although beautiful in structure mathematically, is non-linear in nature. Despite these obstacles, one can be sure that there is indeed some new physics which we are not aware of, at distances close to the Planck length.

On the experimental side, the search for the fundamental length is going on in the Large Hadron Collider at CERN, by studying collisions between particles at ever increasing energies and ultra-precise measurements of various physical properties at low energies. A disagreement between the experimental results and predictions of a quantum electrodynamics or chromodynamics type theory would indicate possible violations of the present concepts of spacetime and the requirement of introducing the fundamental length.